Abstracts

Nolan Wallach: *Generalized Whittaker vectors and Jacquet integrals for parabolically induced representations.*

Let G be a real reductive group and let P be a parabolic subgroup. We will analyze the generalized Whittaker vectors for induced representations from P to G. We prove holomorphic continuations of the integrals that converge in a tube on a traslation of the positive Weyl chamber and analogues of multiplicity one theorems. Our techniques constrain us to a specific class of parabolic subgroups. This class contains most of the arithmetically important ones. We will in addition to a description of the theorems give a classification of the parabolic subgroups to which the theorems apply (joint work with Karin Baur).

Jacques Faraut: *Fock models for minimal representations.*
(Joint work with Dehbia Achab)

Starting from a semi-simple Jordan algebra V of rank ≤ 4, a generalized Kantor- Koecher-Tits construction leads to a 5-graded simple Lie algebra g. A minimal representation of a Lie group G with Lie(G) = g is realized on a Hilbert space H of holomorphic functions on a complex conical manifold whose quotient by dilations is the conformal compactication of V , and which is related to the minimal nilpotent orbit in g. The reproducing kernel of H is computed. This work is inspired by the papers of R. Brylinski and B. Kostant about this topic.

Erik van den Ban: *Paley-Wiener theorems for groups and symmetric spaces.*

We will review a number of Paley-Wiener theorems for semisimple Lie groups and symmetric spaces of the non-compact type, both for compactly supported smooth functions and for distributions. We will also discuss the relation between the so-called Arthur - Campoli conditions and the intertwining conditions of Delorme - Zhelobenko. The latter part of the talk is based on recent joint work with Sofien Souaifi.

Birgit Speh: *Cuspidal representations of reductive groups.*

Let G be a connected reductive algebraic group defined over $\mathbb{Q}$ so that $G(\mathbb{R})$ is noncompact. Let $\tau : G \to G$ be a $\mathbb{Q}-$rational automorphism of finite order. The automorphism acts on the cuspidal automorphic functions on $G(\mathbb{A})$. If F is a finite dimensional representation of $G(\mathbb{R}) \rtimes \{1, \tau\}$, then $\mathrm{tr} F(\tau)$ is well defined. The main result in my talk is the following

Theorem
Let G be a connected reductive linear algebraic group defined over $\mathbb{Q}$. We assume that $G(\mathbb{R})$ has no compact factors and that its derived group is simple. Let F be a finite dimensional irreducible representation of $\mathbb{G}(\mathbb{R}) \times \{1, \tau\}$, and assume that the centralizer of τ in $G(\mathbb{R})$ is of equal rank. If tr $F(\tau) \neq 0$, then there exists a cuspidal automorphic representation $\pi_{\mathbb{A}}$ of $G(\mathbb{A})$ stable under τ, with the same infinitesimal character as F.

In the special case of $G = GL_n$ we consider the involution τ_c with fixed points $SO(n)$, and if $n = 2m$ also the the symplectic involution τ_s with fixed points $Sp(n)$. The following theorem summarizes our results for these special cases.

Theorem
There exist cuspidal representations $\pi_{\mathbb{A}}$ of GL(n,$\mathbb{A}$) with trivial infinitesimal character invariant under the Cartan involution τ_c. If n= 2m there also exist cuspidal representations $\pi_{\mathbb{A}}$ of GL(n,$\mathbb{A}$) with trivial infinitesimal character invariant under τ_s.

All results are joint work with Dan Barbasch.

Patrick Delorme: *A Paley-Wiener theorem for Whittaker functions on a reductive p-adic group.*

We define a Fourier transform for functions on a reductive p-adic group, which transforms by a nondegenerate character of a maximal unipotent subgroup, and with compact support moduli this unipotent. This transformation, as well as wave packets are studied using a theory of the constant term. Then, a result of Heiermann is used to characterize the image of this Fourier transform.

Gregory Landweber: *Off-shell representations of the super Poincaré group.*

The super Poincare group is at the heart of supersymmetry in physics, extending the Poincare group of isometries of Minkowski space to include odd symmetries that are usually interpreted as symmetries of superspace. The unitary representations of the (super) Poincaré group are spaces of sections of homogeneous bundles over covector orbits called mass shells. In contrast to these "on-shell" representations, we investigate "off-shell" representations consisting of unconstrained fields. We will present an alternative to the usual superspace methods of constructing off-shell representations, which work only in low spacetime dimensions, instead interpreting off-shell representations as filtered Clifford supermodules.

Michel Duflo: *Weyl's functional calculus and equivariant differential forms.*
Let $A_1, A_2, \ldots, A_d$ be d Hermitian matrices of size n. Weyl's functional calculus is a compactly supported distribution W on $\mathbb{R}^d$ which associates to a smooth function f of d variables a matrix $W(f) := f(A_1, \ldots, A_n)$. Forty years ago, Edward Nelson gave a formula for W, explicitly describing it as the derivative of a probability measure on $\mathbb{R}^d$ supported on the *joint numerical range* of the A_i. We show how this formula fits in the setting of Hamiltonian geometry and equivariant differential forms.

Bent Orsted: *Representation theory and functional determinants on spheres.*
For elliptic operators such as the Laplace operator or the square of the Dirac operator one may via the asymptotics of the heat kernel define the so-called functional determinant of such an operator. We study functionals of this type on spheres using methods of representation theory, and in particular the nature of stationary points, varying the metric; the Hessian may in many cases be identified as a standard intertwining operator. This is joint work with Niels Martin Møller.

Pavle Pandzic: *Dirac cohomology of Harish-Chandra modules.*
In the 1970's, Parthasarathy introduced a version of the Dirac operator D attached to a real reductive group, and used it to construct the discrete series representations. He also obtained a useful necessary condition, Dirac operator inequality, for unitarizability of an irreducible Harish-Chandra module. In 1997 Vogan studied a purely algebraic version of D and used it to attach an invariant, called Dirac cohomology, to a Harish-Chandra module X. He conjectured that Dirac cohomology, if nonzero, determines the infinitesimal character of X. This conjecture was proved by Huang and myself in 2002. Subsequent generalizations to other settings were obtained by Kostant, Kumar, Alekseev-Meinrenken and Kac-Frajria-Papi. Further results on Dirac cohomology of Harish-Chandra modules included a relationship to n cohomology in some special cases (joint with Huang and Renard). In this talk I will give a brief overview of the definitions and the above mentioned results. I will then describe some further work in progress. The topics I plan to mention are algebraic Dirac induction (with D. Renard), p^+ cohomology of unitary highest weight modules (with V. Protsak) and unipotent representations (with D. Barbasch), and sharpening the Dirac inequality (with D. Renard).

Alexander Lubotzky: *Simple groups of Lie type as expander graphs.*

Based on works of Kassabov, Nikolov, Hadad and the speaker, we will show how all finite simple groups of Lie type (with thepossible exception o fthe Suzuki groups) can be made into expander graphs in a uniform way. The proof starts with SL(2) where works of Drinfeld and Selberg on automorphic forms are used and then extended to all other groups of Lie type by combining (elementary) K-theory, group theory and model theory.

Rajagopalan Parthasarathy: *Representation theoretic harmonic spinors for coherent families.*

Coherent continuation π_2 of a representation π_1 of a semisimple Lie algebra arises by tensoring π_1 with a finite dimensional representation F and projecting it to the eigenspace of a particular infinitesimal character. Some relations exist between the spaces of harmonic spinors (involving Kostant's cubic Dirac operator and the usual Dirac operator) with coefficients in the three modules. For the usual Dirac operator we illustrate with the example of discrete series; more generally, a similar relationship can be seen for cohomological representations by using the construction of generalized Enright-Varadarajan modules. This is joint work with S. Mehdi.

Pierre Pansu: *Flexibility of surface groups in semi-simple Lie groups.*

Say a non Zariski dense homomorphism of a surface group into a Lie group is flexible if it is a limit of Zariski dense homomorphisms, locally rigid otherwise. We survey (global) rigidity results concerning surface groups in semisimple Lie groups, and complement them with new flexibility results.

Jean-Philippe Anker: *Evolution equations in negative curvature.*

We are interested in the heat equation, the Schrödinger equation and the wave equation on noncompact Riemannian symmetric spaces and related structures, such as affine buildings. In this talk we shall mostly discuss heat kernel estimates and the (semi)linear Schrödinger equation, starting with the most simple case, namely real hyperbolic spaces. Our results have been obtained in joint works with P. Ostellari, Br. Schapira and B. Trojan, V. Pierfelice and M. Vallarino.

Leticia Barchini: *Certain Components of the Springer Fiber for Classical Groups.*

This talk is centered around a new approach to the study of the geometry of certain irreducible components of Springer fibers. Springer fibers are the fibers of the moment map of the cotangent bundle of a flag variety. The components under study are known to encode deep information about irreducible admissible representations. The explicit geometric description of these components yield the computation of the associated cycle of representations in the discrete series.

Roberto Miatello: *Lattice points for Hilbert modular groups.*

We will study the counting function for lattice points for Hilbert modular groups by spectral methods, obtaining a main term and an error term that depends on the exceptional spectrum. We will compare with other known results in the subject.

Emmanuel Ullmo: *Équidistribution de mesures adèliques sur les espaces homogènes.*

Werner Müller: *The Selberg trace formula and dynamical zeta functions.*

We consider a compact locally symmetric space $\Gamma\backslash G/K$ and an arbitrary, in particular non-unitary, finite-dimensional representation of Γ. For this set up we establish a twisted version of the Selberg trace formula. We use this trace formula in the rank one case to study twisted Selberg - and Ruelle zeta functions.

Roger Zierau: *The structure of certain components of Springer fibers.*

Components of Springer fibers associated to closed K-orbits in the flag variety of $SU(p,q)$ will be discussed. An explicit description will be given. It will be shown how this description can be used to study the geometry of these components. For example we will see that they are isomorphic to iterated bundles, and are therefore smooth.

Dan Barbasch: *Stable combinations of characters of unipotent representations.*

In his work, Arthur gives a conjectural description of the residual spectrum of automorphic forms. This motivated introducing the notion of special unipotent representations for real reductive groups; representations with maximal annihilator in the universal enveloping algebra, and particular infinitesimal character obtained from a nilpotent adjoint orbit in the dual Lie algebra. Forming stable combinations of such representations is important for applications of the Arthur Selberg trace formula. In the book "Langlands classification and irreducible characters for real reductive groups", we use geometric methods to construct stable combinations of characters. In this talk I will generalize these methods so as to give a basis of the space of stable combinations of unipotent representations. This is based on joint work with Peter Trapa.

Toshiyuki Kobayashi: *Global geometry on locally symmetric spaces – beyond the Riemannian case.*

The local to global study of geometries was a major trend of 20th century geometry, with remarkable developments achieved particularly in Riemannian geometry.
In contrast, in areas such as Lorentz geometry, familiar to us as the space-time of relativity theory, and more generally in pseudo-Riemannian geometry, as well as in various other kinds of geometry (symplectic, complex geometry, ...), surprising little is known about global properties of the geometry even if we impose a locally homogeneous structure.
In this talk, I plan to give an exposition on the recent developments on the question about the global natures of locally non-Riemannian homogeneous spaces, with emphasis on the existence problem of compact forms, rigidity and deformation.

T.N. Venkataramana: *TBA.*

Leslie Saper: *L-modules and the cohomology of locally symmetric spaces.*

The theory of L-modules was developed to solve the conjecture of Rapoport and Goresky-MacPherson: the intersection cohomology of the Baily-Borel-Satake compactification of a Hermitian locally symmetric space is isomorphic to the intersection cohomology of the reductive Borel-Serre compactification. However it applies more generally and is an powerful combinatorial tool to study constructible sheaves on the reductive Borel-Serre compactification of a general locally symmetric space. We will survey the theory and give applications to several areas, including cohomology of arithmetic groups, L^2-cohomology, L^2-harmonic forms, and weighted cohomology.

Birne Binegar: *HC-Cells, Nilpotent Orbits, Primitive Ideals, and Weyl Group Representations.*

Let G be the real points of linear reductive complex algebraic group defined over $\mathbb{R}$, let $\hat{G}_\lambda$ be the set of equivalences classes of irreducible admissible representations of G of regular integral infinitesimal character lambda. The Atlas software enumerates the representations in $\hat{G}_\lambda$, and computes the Kazhdan-Lusztig-Vogan polynomials which not only prescribe the Jordan-Holder decomposition of standard modules of infinitesimal character lambda in terms of the irreducibles in $\hat{G}_\lambda$, but also can be employed to endow the set $\hat{G}_\lambda$ with the structure of a W-graph, a certain weighted directed graph replete with valuable representation theoretical data. The strongly connected components of this W-graph are the HC-cells of $\hat{G}_\lambda$. In this talk I will describe how the induced W-graph structure of an HC-cell allows one to compute the nilpotent orbit attached to a primitive ideal of the annihilator of representation, allows one to determine when two representations in $\hat{G}_\lambda$ share the same primitive ideal, and in general provides one with a vital instrument for displaying the connections between irreducible representations, primitive ideals, nilpotent orbits, and Weyl group representations.

David Vogan: *Branching laws and signature for real reductive groups.*

Suppose G is a real reductive Lie group, and K is a maximal compact subgroup. If X is an irreducible (admissible) representation of G, then the restriction to K decomposes as a discrete direct sum of irreducible representations of K, with finite multiplicities: X restricted to K equals

$$\sum_{\mu \in \hat{K}} m(\mu, X)\mu,$$

with $m(\mu, X)$ a non-negative integer. The problem of computing the function m is one of the most fundamental in representation theory; it was solved more than twenty-five years ago by work of Hecht-Schmid and Kazhdan-Lusztig, building on a wonderful theory developed by Harish-Chandra and Langlands.

If X carries an invariant Hermitian form, then the signature of this form defines non-negative integer-valued functions p and q with

$$p(mu, X) + q(mu, X) = m(mu, X).$$

A great deal is known about the functions p and q, but we are still not able to compute them in general. The problem of deciding when q is equal to zero is the problem of determining the unitary dual of G.

I will describe recent progress of Marc van Leeuwen and the (Atlas of Lie groups project) toward software to compute $m(mu, X)$. At the same time I will discuss what is understood about the functions p and q.